A Predictive Minimal Model for Neutrino Masses and Mixings

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A model is considered in which the scale of the heavy singlet neutrinos is a few orders of magnitude below the grand unification scale and where right-handed vector bosons play still a negligible role. In a basis with diagonal up-quark and Dirac-neutrino mass matrices it is assumed that the heavy neutrino mass matrix has only zero elements in its diagonal, in analogy to the light neutrino mass matrix in the Zee model. Connecting then the remaining matrix elements with the small parameter describing the hierarchy of quark masses and mixings and by assuming commutativity of the charged lepton with the down-quark mass matrix, the calculation of all neutrino properties can be performed in terms of the two mass differences relevant for atmospheric and solar neutrino oscillations. CP-violation is directly related to CP-violation in the quark sector.

I. INTRODUCTION

Recent experiments give strong evidence that neutrinos oscillate and have finite masses [1], [2]. These experiments provide the first conclusive evidence for physics beyond the standard model. Good candidates for a corresponding extension of the standard model are supersymmetric or non-supersymmetric grand unified theories [3]. For recent summaries in which many models are discussed and for further literature see [4]. Detailed grand unification models need many assumptions about numerous new particles and their representations, Higgs potentials and couplings. One obtains interesting relations, but it is often hard to see to what extent the many assumptions are really necessary and decisive for the light neutrino sector. In this article I will formulate a model in which the heavy neutrinos get masses several orders of magnitude below the scale of grand unification. The possibility exists that these masses are generated dynamically in some analogy to the generation of light neutrino masses in the Zee model [5]. Only few particle properties at the unification scale are then directly relevant for the neutrino problem. The minimal model obtained in this way provides an example in which the significance of the assumptions are obvious and the neutrino properties can be calculated in all details. A disadvantage compared to an explicit treatment of a specific grand unified theory is that there are no immediate additional predictions about new physics at the unification scale.

Oscillations involving 3 light neutrinos[1] can be described in terms of 6 parameters: 2 parameters for the difference between the square of the neutrino masses and 4 parameters for the unitary matrix U which relates – in a basis in which the charged lepton mass matrix is diagonal – the flavor eigenstates ν_e, ν_μ, ν_τ with the mass eigenstates ν_1, ν_2, ν_3

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \tag{1}$$

The theoretical understanding of neutrino physics and of these parameters in particular requires, however, the consideration of many more – a priori unknown – quantities. An appealing way to extent the standard model is to add to the 3 two-component fields new two-component fields $\hat{\nu}_e, \hat{\nu}_\mu, \hat{\nu}_\tau$, which are singlets with respect to the standard model gauge group and acquire large masses corresponding to the scale of the new physics involved. The well-known see-saw mechanism then provides for the observed smallness of the light neutrino masses [4].

There are then 3 mass matrices to consider[2]: the Dirac neutrino mass matrix m_ν^{Dirac} which connects the old with the new two-component fields, the mass matrix for the charged leptons m_E and the mass matrix of the singlet neutrinos M_R.

There is some freedom for choosing a basis for the 3 matrices: We may take the Dirac neutrino mass matrix diagonal and real which leaves us with 3 parameters for this matrix. Below the scale of new physics the charged lepton mass matrix can be taken hermitian. After a proper adjustment of the phases of the lepton fields this matrix contains 7 parameters. Finally, the symmetric mass matrix M_R can be transformed to have real diagonal elements and contains, therefore, 9 parameters.

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[1]In this article the possibility of the existence of light sterile neutrinos is not discussed even though they are needed in a general analysis, in particular, if the finding of the LNSD group [6] are included.

[2]A classification of three-neutrino models dealing with these mass matrices is provided in ref. [7]. Ansätze related to the present approach can be found in ref. [8]. I will closely follow and extend my previous publications [9] on this subject.

In the next sections we will restrict these 19 parameters using the ideas mentioned above and by suggesting connections with the known structure of the mass matrices of up and down quarks. The see-saw formula

$$m_\nu = -m_\nu^{Dirac} \cdot M_R^{-1} \cdot (m_\nu^{Dirac})^T \tag{2}$$

together with the mass matrix for charged leptons then allows to calculate all the properties of light neutrinos of interest here.

II. THE DIRAC NEUTRINO AND THE UP-QUARK MASS MATRICES

Below the scale of new physics we are free to choose a basis in which the Dirac neutrino mass matrix m_ν^{Dirac} and the up-quark mass matrix m_U are diagonal simultaneously. In grand unified theories (taking low Higgs representations) one finds that at the unification scale m_U and m_ν^{Dirac} are closely related and could even be equal there [10]. We will assume, therefore, that at the mass scale of the heavy neutrinos, which will turn out to be much lower than the unification scale, m_ν^{Dirac} and m_U can be diagonalized simultaneously. In section 4 we will show that this mass scale which we denote by M_0 is of order 10^{11} GeV.

The observed hierarchical structure of charged fermion masses and mixings allows to express the corresponding mass matrices in terms of powers of a small quantity. Using for this parameter $\sigma = (m_c/m_t)^{1/2}$, the diagonal up-quark mass matrix can be written [9]

$$m_U(M_0) = \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_t(M_0)$$

$$\sigma = 0.057 \qquad . \tag{3}$$

With the help of the renormalization group equations Eq. (3) together with the down-quark and charged lepton mass matrices (see section 5) provides values for the masses of up, charm and top quarks at the scale of the vector boson Z which agree with the known result [11]

$$m_u(m_Z) = 1.9 \pm 0.4 \text{ MeV}, \qquad m_c(m_Z) = 0.61 \pm 0.05 \text{ GeV},$$
$$m_t(m_Z) = 173 \pm 5 \text{ GeV},$$
$$m_d(m_Z) = 3.4 \pm 0.6 \text{ MeV}, \qquad m_s(m_Z) = 0.064 \pm 12 \text{ GeV},$$
$$m_b(m_Z) = 2.90 \pm 0.04 \text{ GeV} \tag{4}$$

within the given error limits. For m_ν^{Dirac} at the scale M_0 we can write therefore

$$m_\nu^{Dirac}(M_0) = \begin{pmatrix} y\sigma^4 & 0 & 0 \\ 0 & x\sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_t(M_0) \tag{5}$$

with $x = O(1)$, $y = O(1)$. There is no point in taking an additional $O(1)$ parameter for the 33 element. As can be seen below this would only affect the scale parameter M_0 which is a fit parameter in our model.

III. THE MASS MATRIX FOR THE HEAVY SINGLET NEUTRINOS

In the absence of a detailed theory describing new physics beyond the standard model, one can only speculate about the mass matrix for the heavy neutrinos. However, one can hope that the small parameter σ which governs the up and down quark mass matrices plays also here an important role [9]. Because of the self-coupling of the heavy neutrinos the assignment of generation quantum numbers to those fields [12] gives additional restrictions for the powers of σ occurring in this matrix. As an Ansatz for M_R I will use here a matrix with only zero elements in its diagonal (see also [13]), in analogy to the matrix for the light neutrinos obtained in the Zee model [5]. M_R could have a similar origin, this time, however, involving the "right-handed" fields in a right-left symmetric theory such as $SO(10)$ or $E6$ at the grand unification scale $\gg M_0$. The elements of a matrix of this form can easily be connected with the elements of the mass matrix m_ν^{Dirac} (and m_U) with the powers of σ related to appropriate generation charges of the singlet neutrino fields. Therefore, I propose

$$M_R = \begin{pmatrix} 0 & x\sigma^3 & \sigma \\ x\sigma^3 & 0 & a \\ \sigma & a & 0 \end{pmatrix} M_0 \tag{6}$$

with $a = O(1)$. In this Ansatz, the ratio between $(M_R)_{12}$ and $(M_R)_{13}$, namely $x\sigma^2$, has been taken to be identical to the ratio between the elements $(m_\nu^{Dirac})_{22}$ and $(m_\nu^{Dirac})_{33}$ in order to have a close relation with m_ν^{Dirac}. Since this ratio is a real number, all elements of M_R can be taken to be real by a proper phase choice for the singlet neutrino fields. The powers of σ occurring in (6) correspond to generation charges for the fields $\hat{\nu}_e, \hat{\nu}_\mu, \hat{\nu}_\tau$ equal to $5/2, 3/2, -1/2$, respectively, when setting the generation charge of the scalar field which produces the heavy masses equal to -1.

M_R can be diagonalized by an orthogonal matrix V_R defined by

$$M_R = V_R M_R^{diagonal} V_R^T \quad . \tag{7}$$

The two large eigenvalues of M_R differ up to order σ^2 only in sign (like the eigenvalues of a Dirac neutrino mass matrix) ($\simeq \pm a M_0$) while the third eigenvalue is smaller by the factor σ^4. Because of the almost degeneracy of two eigenvalues it is interesting to look at the up-quark mass matrix when taken in a basis in which M_R is diagonal. One finds – up to order σ^2 – for $x = 1$

$$V_R m_U V_R^T = \begin{pmatrix} \frac{\sigma^2}{2a^2} & -\frac{\sigma}{2a} & -\frac{\sigma}{2a} \\ -\frac{\sigma}{2a} & \frac{1}{2} + \frac{\sigma^2}{2}(1 - \frac{1}{a^2}) & \frac{1}{2} - \frac{\sigma^2}{2}(1 + \frac{1}{4a^2}) \\ -\frac{\sigma}{2a} & \frac{1}{2} - \frac{\sigma^2}{2}(1 + \frac{1}{4a^2}) & \frac{1}{2} + \frac{\sigma^2}{2} \end{pmatrix} m_t(M_0). \tag{8}$$

Obviously, to a very good approximation, this matrix is of the "democratic" form in the 2,3 sector.

IV. THE SEE-SAW NEUTRINO MASS MATRIX

The see-saw neutrino mass matrix m_ν can now be obtained from Eq. (2) using (5) and (6). But before, we rescale the parameters a and M_0 according to $a \to \frac{x}{y}a$, $M_0 \to y M_0$. One then gets the same expression for the light neutrino mass matrix m_ν as in the special case in which x and y in (5) and (6) are equal to 1:

$$m_\nu(M_0) = -\frac{\sigma^2}{2aM_0} \begin{pmatrix} -a^2\sigma^2 & a\sigma & a\sigma \\ a\sigma & -1 & 1 \\ a\sigma & 1 & -1 \end{pmatrix} (m_t(M_0))^2 \tag{9}$$

The basis chosen is still the basis in which m_U and m_ν^{Dirac} are diagonal matrices. Thus, we are not yet in a basis in which the charged lepton mass matrix is diagonal. We also have to use the renormalization group equation to go from the scale M_0 down to the scale of the Z boson. But one can calculate from (9) the eigenvalues of the light neutrinos at M_0. As we will see below, the pattern of these eigenvalues will not change on the way down.
The eigenvalues are

$$m_1(M_0) = -\frac{(m_t(M_0))^2}{M_0}\left(\frac{\sigma^3}{\sqrt{2}} - \frac{a\sigma^4}{4}\right)$$

$$m_2(M_0) = \frac{(m_t(M_0))^2}{M_0}\left(\frac{\sigma^3}{\sqrt{2}} + \frac{a\sigma^4}{4}\right)$$

$$m_3(M_0) = \frac{(m_t^2(M_0))^2}{M_0}\frac{1}{a}\sigma^2 \tag{10}$$

One can now identify the mass differences $m_3^2 - m_1^2 \approx m_3^2 - m_2^2$ with Δm^2 observed in atmospheric neutrino experiments. The Super-Kamiokande result [2] $\Delta m^2 \approx 3 \cdot 10^{-3} (\text{eV})^2$ gives for aM_0:

$$aM_0 \approx 5 \cdot 10^{11} \text{ GeV} \tag{11}$$

For the ratio of the difference of squared masses, one gets

$$\delta = \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} = \frac{a^3\sigma^3}{\sqrt{2}} + O(\sigma^5) \tag{12}$$

Thus, the mass difference $m_2^2 - m_1^2$ relevant for solar neutrino oscillations turns out to be

$$m_2^2 - m_1^2 \approx 4 \cdot 10^{-7} a^3 \ (\text{eV})^2. \tag{13}$$

Interestingly, we are left with the single parameter a only.

As long as the off-diagonal elements of the charged lepton mass matrix are not much larger in magnitude than the off-diagonal elements of the down-quark mass matrix (see the next section), the diagonalization of (9) provides already an estimate for the neutrino mixing matrix U. The orthogonal matrix O_M which diagonalizes $m_\nu(M_0)$ is

$$O_M = \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{a\sigma}{8} & -\frac{1}{\sqrt{2}} - \frac{a\sigma}{8} & 0 \\ \frac{1}{2} + \frac{a\sigma}{8\sqrt{2}} & \frac{1}{2} - \frac{a\sigma}{8\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} + \frac{a\sigma}{8\sqrt{2}} & \frac{1}{2} - \frac{a\sigma}{8\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(\sigma^2) \tag{14}$$

Because $\sigma/8$ is a very small number, one has the remarkable result that the neutrino mixing matrix is of the bimaximal type [14] and very little dependent on the mass difference of the 2 lightest neutrinos in the range acceptable for solar neutrino oscillations [1,4].

V. THE CHARGED LEPTON AND THE DOWN-QUARK MASS MATRICES

We know the eigenvalues of the charged lepton mass matrix from experiment but not its form in a basis in which the Dirac neutrino and the up-quark mass matrices are diagonal. The down-quark mass matrix on the other hand is known to a good extent from the measured down-quark masses and the Cabibbo-Kobayashi-Maskawa mixing angles. In grand unified theories one obtains relations between m_E and m_D [10], [15], but the details depend on the Higgs representations and Yukawa couplings. The well-known relation obtained in $SU(5)$ [3], [16]

$$m_E^T \approx m_D \tag{15}$$

cannot be exact because of the different mass ratios in the quark and lepton sector. Below the scale M_0 both m_E and m_D can be taken to be hermitian matrices. In the model considered here this should still be possible when reaching the scale M_0, which is much smaller than the unification scale where right-handed vector bosons are effective. Otherwise we would have to introduce more structure and thereby more unknown parameters. From the relations in grand unified theories such as (15) one can then conclude that the mixing for charged leptons are similar to the mixing of quarks, i.e. not very large[3]. The mixing matrix U for neutrinos (at M_0) will then not much differ from O_M given in (14).

To be more specific I will consider the hypothesis that at M_0, m_E^T and m_D can be diagonalized simultaneously. In other words, in our model both matrices, if put into hermitian forms, commute as do the matrices m_ν^{Dirac} and m_U [9].

$$[m_E^T, m_D] = 0 \quad . \tag{16}$$

On the one hand, this condition is weaker than (15) since the matrix elements of m_E^T and m_D do not need to be equal or nearly equal. On the other hand, it is more strict since (16) allows a complete calculation of m_E from the lepton masses and m_D (or the CKM matrix). Of course all this has to be done at M_0 and should then be scaled down by the renormalization group equations [19] to the weak scale, i.e. the scale of the Z-boson. For $m_D(M_0)$ we choose the matrix

$$m_D(M_0) = \begin{pmatrix} 0.7\sigma^3 & i1.54\sigma^2 & -i\sigma^2 \\ -i1.54\sigma^2 & -\sigma/3 & i0.8\sigma \\ i\sigma^2 & -i0.8\sigma & 1 \end{pmatrix} m_b(M_0) \quad , \tag{17}$$

which together with m_U of Eq. (3) leads to mass eigenvalues at the weak scale within the experimental uncertainties (4). It also gives at m_Z

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[3]In this respect the present approach differs decisively from models which use asymmetric matrices by proposing for the quark sector large "right-handed" mixings of physical relevance correlated with a large mixing in the "left-handed" lepton sector [17], [18].

$$|V_{ud}| = 0.22, \quad |V_{cb}| = 0.039, \quad |V_{ub}| = 0.0032 \quad . \tag{18}$$

The phases chosen in (17) are such as to obtain "maximal" CP-violation [15,9] in the quark sector which may or may not be supported by ongoing experiments.

An important matrix depending on the off-diagonal elements of m_D and which is relevant for the amount of CP violation in the quark sector, is the commutator [20]

$$[m_U, \ m_D] = \frac{1}{i} C_q(M_0) \quad . \tag{19}$$

The analog commutator for leptons is

$$[m_\nu^{Dirac}, \ m_E^T] = \frac{1}{i} C_\ell(M_0) \quad . \tag{20}$$

As an alternative to (16) one can require that the off-diagonal elements of m_D and m_E have the same origin, i.e. arise from the same Higgs field:

$$C_\ell(M_0) = \frac{m_\tau(M_0)}{m_b(M_0)} C_q(M_0) \quad . \tag{21}$$

The condition (21), with $x = 1$, m_D from (17), and the known charged lepton masses allows another calculation of $m_E(M_0)$

$$m_E(M_0) = \begin{pmatrix} -0.66\sigma^3 & -i1.54\sigma^2 & i\sigma^2 \\ i1.54\sigma^2 & -\sigma & -i0.8\sigma \\ -i\sigma^2 & i0.8\sigma & 1 \end{pmatrix} m_\tau(M_0) \tag{22}$$

This matrix differs somewhat from the one obtained from (16) but leads again to small charged lepton mixing angles.

It would be interesting if another basis-independent relation between the mass matrices would exist, one which involves the lepton mass matrices only. Since M_R has no diagonal elements in our model one could speculatively assume

$$[m_\nu^{Dirac}, \ m_E^T] = \frac{1}{i} \frac{m_t(M_0)m_\tau(M_0)}{M_0} \sigma M_R(M_0) \quad . \tag{23}$$

Also this relation could be used to calculate m_E. The result, although somewhat different from the one obtained for m_E in (22) fixes the phases of m_E and leads again to charged lepton mixing angles of similar small magnitudes.

VI. THE CALCULATION OF THE NEUTRINO MASSES AND THE NEUTRINO MIXING ANGLES

At the scale M_0 the mass matrix for light neutrinos is taken from Eq. (9). After the diagonalization of the mass matrix for charged leptons at this scale

$$m_E(M_0) = U_E(M_0)m_E^{diagonal}(M_0) \ U_E^\dagger(M_0) \ , \tag{24}$$

$m_\nu(M_0)$ has to be transformed accordingly. The new neutrino mass matrix is then

$$\tilde{m}_\nu(M_0) = (U_E(M_0))^T m_\nu(M_0) \ U_E(M_0) \quad . \tag{25}$$

This new neutrino matrix has now complex elements and will lead therefore to CP-violation effects in neutrino processes. $\tilde{m}_\nu(M_0)$ can now be scaled down to the weak scale by the corresponding renormalization group equation. The evolution equation according to the standard model is [21]:

$$(4\pi)^2 \frac{d}{dt}\tilde{m}_V = (-3g_2^2 + 2\lambda)\tilde{m}_\nu$$
$$+\frac{4}{v^2}\text{Tr}[3m_U m_U^\dagger + 3m_D m_D^\dagger + m_E m_E^\dagger]\tilde{m}_\nu$$
$$-\frac{1}{v^2}(\tilde{m}_\nu m_E m_E^\dagger + (m_E m_E^\dagger)^T \tilde{m}_\nu) \quad . \tag{26}$$

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$\lambda = \lambda(t)$ denotes the Higgs coupling constant related to the Higgs mass according to $m_H^2 = \lambda v^2$ with $v = 246$ GeV. We take $m_H(m_Z) = 150$ GeV for the numerical calculation. Solving (26) allows to obtain the neutrino mass matrix $\tilde{m}_\nu$ at the scale of the standard model. The neutrino mixing matrix $U = U(m_Z)$ can then be obtained by diagonalizing the hermitian matrix $\tilde{m}_\nu^* \tilde{m}_\nu$:

$$\tilde{m}_\nu^*(m_Z)\, \tilde{m}_\nu(m_Z) = U(\tilde{m}_\nu^* \tilde{m}_\nu)^{diagonal} U^\dagger. \tag{27}$$

The diagonal matrix

$$\tilde{m}_\nu^{diagonal} = U^T \tilde{m}_\nu(m_Z) U \tag{28}$$

then provides the (complex) neutrino mass eigenvalues. By introducing the diagonal phase matrix ϕ which consists of the phase factors of $\tilde{m}_\nu^{diagonal}$ divided by 2, U can be redefined $U \to U\phi$. The phase factors in (28) cancel and the so obtained unitary matrix U expresses the neutrino states ν_e, ν_μ, ν_τ by the neutrino mass eigenstates according to Eq. (1).

The change of the mass matrices m_U, m_D, m_E and $\tilde{m}_\nu$ between m_Z and M_0 depends, of course, on the renormalization group equations. To find the expression (17) for $m_D(M_0)$ from the – approximate – knowledge of $m_D(m_Z)$ I used the renormalization group matrix equations of the standard model [19]. It leads to a change of $|V_{cb}|$ by about 10%. When using the renormalization group equation of the minimal supersymmetry model [19], the main difference to the standard model case lies in the overall factors $m_b(M)$, $m_t(M)$, which can easily be adjusted. For $\tilde{m}_\nu(M)$ the formula (26) has to be replaced by the corresponding supersymmetry formula given by Babu et al. [21].

VII. RESULTS AND DISCUSSION

It is straightforward to calculate from (9),(16),(17),(24)-(28) the light neutrino masses and the unitary mixing matrix U in terms of the parameters M_0 and a. In a very direct way these two parameters determine the 2 mass differences in the 3 neutrino scenario. For the evolution of the mass matrices m_U, m_D, m_E and $\tilde{m}_\nu$ as a function of the scale we applied the renormalization group equations according to the standard model and according to the minimal supersymmetry model. It turned out that for fixed M_0 and a there is no noticeable change in the ratio of the 3 eigenvalues of $\tilde{m}_\nu$ when going from M_0 down to m_Z. Even when the squared mass difference of the two lightest neutrinos is taken to be very small their mass ratio remains unchanged. Since both masses have opposite signs as seen in (10), radiative corrections are ineffective as proved in ref. [22].

Interestingly, it also turned out that the neutrino mixing matrix U is practically independent of the scale parameter. Moreover, the model predicts U to depend only very little on the difference of the neutrino masses. The reason is that in the mixing matrix (14) the parameter a plays a minor role only. Because also the mixing angles of the charged leptons are small as described in section 5, the matrix U of our model is of the bimaximal form [14] for all mass differences of interest in neutrino oscillation experiments[4].

The deviations from bimaximal mixing are therefore almost exclusively due to the mixings of charged leptons in the basis in which m_ν^{Dirac} is diagonal. Using for $m_E(M_0)$ the result obtained from (16) and (17), one gets

$$Abs[U(m_Z)] = \begin{pmatrix} 0.69 & 0.71 & 0.15 \\ 0.51 & 0.51 & 0.69 \\ 0.51 & 0.49 & 0.71 \end{pmatrix} \tag{29}$$

i.e. nearly pure bimaximal mixing. The amount of CP violation depends evidently on the corresponding CP-violating phase in the quark sector. With the phase choice taken in (17), which at the weak scale provides an acceptable form of the conventional unitarity triangle for quarks, one obtains a neutrino unitarity triangle with angles:

$$\alpha_\nu \approx 77^o, \quad \beta_\nu \approx 17^o, \quad \gamma_\nu \approx 86^o \quad . \tag{30}$$

The form of this triangle is sensitive to the amount of CP violation in the quark sector. When using instead of (16) the condition (21) i.e. $m_E(M_0)$ from (22), (or from the condition (23)) the triangle becomes more flat $(\alpha_\nu \approx 83^o, \quad \beta_\nu \approx 6^o, \quad \gamma_\nu \approx 91^o)$ and $|U_{e,3}| \approx 0.06$.

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[4]One may note here that for a small value of a, $a \approx \sigma$, which describes a mass squared difference between the lightest neutrinos of order $10^{-10}(\text{eV})^2$, m_ν in (9) takes a form not much different from the neutrino mass matrix suggested in [9] and gives similar results.

In neutrino-less double β decay experiments [24] one measures the neutrino mass averadge $|<\tilde{m}>_{e,e}|$. The model predicts the small value

$$|<\tilde{m}>_{e,e}| \approx 0.002 \ eV \ .\tag{31}$$

As we have seen, the model presented here is predictive . Essentially, only the two mass differences between the 3 neutrinos can be choosen freely. It predicts the bimaximal mixing form for the neutrino mixing matrix independently of the mass difference responsible for the solar neutrino oscillations. It is a minimal model because the parameter which describes the hierarchy in the quark sector could be used also for the lepton mass matrices, in particular, for the heavy neutrino mass matrix, providing in this way for an intimate connection between quarks and leptons. It is a minimal model also because only the simplest assumptions are made for the connection between m_ν^{Dirac} and m_U and between m_E and m_D. The idea is that specific particles and interactions acting at the grand unification scale, besides producing the heavy neutrinos at M_0, may play a minor role at this lower scale. To give M_R the form of a Zee matrix is, of course, a strong assumption but may seem not unreasonable for dynamically generated singlet neutrino masses 4 to 5 orders of magnitudes below unification. One obtains for the heavy neutrinos two almost degenerate mass eigenstates which are strongly mixed. Accordingly, in the special basis in which M_R is diagonal, the matrices m_U, m_D, m_ν^{Dirac} and m_E are of a "democratic" form in the 2,3 sector (see Eq. (8)). In other words, in this specific frame the Higgs field acts with (almost) equal strength on the members of the second and third family. However, it is worth pointing out that one can relax the Zee ansatz by including a 33 element to M_R. Because the inverse of M_R enters the see-saw formula even a very large 33 element (of order 10^2) still leads to near bimaximal mixing as long as the ratio $(M_R)_{12}/(M_R)_{13}$ used in (6) is kept fixed. Thus, the oscillation properties of the light neutrinos cannot distinguish this strongly hierarchical mass matrix with small mixings of the heavy neutrinos[5] from the mass matrix of Eq. (6).

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